

Is Dark Matter a BEC or Scalar Field?

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This is a brief review on the history of the Bose-Einstein condensate (BEC) or boson star model of galactic dark matter halos, where ultra-light scalar dark matter particles condense in a single BEC quantum state. The halos can be described as a self-gravitating, possibly self-interacting, coherent scalar field. On a scale larger than galaxies, dark matter behaves like cold dark matter while below that scale the quantum mechanical nature suppresses the dark matter structure formation due to the minimum length scale determined by the mass $m \gtrsim 10^{-24} \text{eV}$ and the self-interaction of the particles. This property could alleviate the cusp problem and missing satellite problems of the Λ CDM model. Furthermore, this model well reproduces the observed rotation curves of spiral and dwarf galaxies, which makes the model promising.

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Dark matter (DM) and dark energy [1] are two of the most important unsolved puzzles in modern physics and cosmology. Although it is well known that the flatness of the galactic rotation curves implies the presence of invisible dark matter around galactic halos [2], only a few properties of the dark matter are known so far. Identification of one DM species by a direct detection experiment, such as Large Hadron Collider (LHC) or DAMA [3], is not enough to fully solve the dark matter problem, because there can be multiple species of DM and we have to explain the observed abundance and distribution of DM in the universe.

While the cold dark matter (CDM) with the cosmological constant (Λ CDM) model is popular and remarkably successful in explaining the large-scale structure of the universe, it seems to encounter problems on the scale of galactic or sub-galactic structure. Numerical simulations with the Λ CDM model usually predict a cusped central density and too many sub-halos which are in contradiction with observations [4, 5, 6, 7]. Since typical CDM particles, such as WIMPs, are heavy and slow, they have a tendency to clump forming a smaller structure, such as a dark matter star or dark matter planet, while observations indicate that the smallest DM-dominated structure is a dwarf galaxy. This implies a natural minimal length scale of DM. Furthermore, observed spins of halos seem to be larger than the predictions of the Λ CDM model. Although the significance of this discrepancy is still controversial, we cannot safely ignore these problems. Thus, it is desirable to consider an alternative DM candidate both playing the role of CDM for the scales larger than a galaxy and at the same time, suppressing sub-galactic structures. Fortunately, we already have one, the Bose-Einstein condensate (BEC) or boson star dark matter model of halos [8, 9], which I briefly review in this paper. (More generally, one can also call the boson star model the scalar field dark matter (SFDM) model of halos, as dubbed in Ref. 10)

In 1992, to explain the observed properties of galactic rotation curves, such as flatness and ripples, Sin [8] suggested that galactic halos are astronomical objects in the BEC of ultra-light DM particles such as pseudo Nambu-Goldstone boson (PNGB). In this model, the halos are like gigantic atoms where cold boson DM particles are condensated in a single macroscopic wave function and the quantum mechanical uncertainty principle prevents halos from self-gravitational collapse. The halo is described by a wave function $\psi(r)$ of the non-linear Schrödinger equation (Gross-Pitaevskii equation (GPE)) with Newtonian gravity:

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + GmM_0\int_0^{r'}dr'\frac{1}{r'^2}\int_0^rdr''4\pi r''^2|\psi|^2\psi(r), \quad (1)$$

where m is the DM particle mass and M_0 is the mass of the halo. This is the first BEC model of galactic dark matter halos. According to the model, the condensation of DM particles, whose huge Compton wavelength $\lambda_{\text{comp}} = 2\pi\hbar/mc \sim 10 \text{ pc}$, i.e., $m \simeq 10^{-24} \text{eV}$, is responsible for the halo formation. The formation of DM structures smaller than the Compton wavelength is suppressed by the uncertainty principle. Later it was shown [11, 12, 13, 14] that this property could alleviate the aforementioned problems of the Λ CDM model. Despite of their tiny mass, BEC DM particles behave like CDM particles [15] during the cosmological structure formation because their velocity dispersion

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is very small. The Newtonian rotation velocity of stellar objects at radius r in DM halos is given by

$$v_{rot}(r) = \sqrt{\frac{GM(r)}{r}}, \quad (2)$$

where $M(r)$ is mass within r .

In the same year the author and Koh [9, 16] generalized Sin's BEC model by allowing a repulsive self-interaction among dark matter particles in the context of quantum field theory and general relativity. This is the first boson star (BS) model of dark matter halos (boson halos) in which the dark matter composes a giant boson star described by a coherent complex scalar field ϕ , which has a typical action

$$S = \int \sqrt{-g} d^4x \left[\frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2} \phi_{;\mu}^* \phi_{;\nu} - V(\phi) \right] \quad (3)$$

with a repulsive potential $V(\phi)$ and a spherical symmetric metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega. \quad (4)$$

In the work, a quartic potential $V(\phi) = \frac{m^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi|^4$ was used as an example. In this model, when $\lambda > 0$, the repulsive self-interaction among DM particles, besides the uncertainty principle, can provide an additional mechanism against gravitational collapse. It was also pointed out that one can extend the model to oscillatons, boson-fermion stars, and Q-stars, which were later investigated by others independently [17]. Since the BS model can be reduced to the BEC model in the Newtonian limit and since the observed rotation velocity $v_{rot} \sim 10^{-3}c$, where c is the light velocity, the BS model predicts rotation curves very similar to those of the BEC DM halo model.

To my knowledge, at least for dark matter halos, this BS model is also the first model to use the coherent SFDM [9]. Using the boson star theory, it was pointed out that even a tiny self-coupling of the scalar field could drastically change the scales of dark matter halos and increase the allowable range of the mass of dark matter particles. From the condition that the halo mass, M_{halo} , should be smaller than the critical mass of a BS, it was found that [9]

$$\lambda^{\frac{1}{2}} \left(\frac{M_P}{m} \right)^2 \gtrsim 10^{50}, \quad (5)$$

which leads to $10^{-24} \text{ eV} \lesssim m \lesssim 10^3 \text{ eV}$ for $0 \leq \lambda \leq 1$. Here, the lower bound came from the Compton wavelength for $\lambda = 0$. This is a relation between the mass and the coupling of the halo dark matter particle. Henceforth, I will designate the two models (BEC and BS models) as the BEC/SFDM model for simplicity.

Unfortunately, these two original works were not widely known. Later, similar ideas were rediscovered and developed by many authors using various potentials and fields such as the massless field [18], the non-minimal coupling [19], quintessence [20, 21], the short-range force (Repulsive DM) [22], fuzzy DM [23], fluid DM [11], the nontopological soliton [24], the *cosh* potential [12, 25] and many others [26, 27, 28, 29, 30]. (See Ref. 31 for a brief review.) However, it is clear that the common concept behind all these DM models, i.e., BEC DM halos or SFDM halos, was introduced in the BEC model and the BS model as early as 1992. (Note that a BS had been called by many different names, such as a soliton star, a scalar star, and a Klein Gordon geon [32].) On the other hand, although strongly self-interacting dark matter (SIDM) [33] and the BS model share a property that DM particles may strongly self-interact repulsively, the two models are different in that DM particles in the SIDM model are not in a coherent state and are usually very massive. There is an idea that a vector field in a modified gravity action could be identified with a BEC [34].

The BEC/SFDM model has many good features. In the original works, it was emphasized that this model could explain the flatness and the ripple structure of the rotation curves, which was later verified by observations by many independent researchers [20, 35, 36, 37]. Furthermore, it was later found that these models could be free from the cusp problem [11, 38] and the missing satellite problem [11, 12, 13, 14], which made this model promising. Recent studies on the dark matter in halos of satellite dwarf spheroidal (dSph) galaxies of the Milky Way have attracted much interest [39, 40]. The dSph galaxies seems to be the smallest dark-matter-dominated astronomical objects and, hence, are ideal objects for studying the nature of dark matter. The observation suggest that a typical dSph never has a size less than $R_{halo} \sim 10^3 pc$, and that the central dark matter halo density profile is not cusped. The boson star model of dSphs seems to be also consistent with the analysis of Ref. 40. According to the observations, the halos of dSphs seem to be cored halos with isotropic stellar velocity, and more massive halos have higher central densities. This mass distribution shows a universal profile and implies a large spatial scale length. Interestingly, all these are characteristic of BS halos. The mass of a boson star is proportional to the central density [32] and a BS has a constant core density [42]. Recently, it was also shown that the BEC (automatically, the BS model, too) reproduces the rotation curves of dwarf galaxies very well [43, 44]. Note that no other DM models has successfully reproduce the

observed rotation curves at this level so far. There are the theories considering the role of visible matter to explain the discrepancy between the numerical simulations in the Λ CDM model and the observations, but, these explanations seem to be rather ad hoc and complicated compared to the explanation of the BEC/SFDM model.

Let us discuss the BEC/SFDM model in detail. The cold gravitational equilibrium configurations of a scalar field, called a boson star (See, for example, Refs. 43 and 46), were found by solving the Klein-Gordon equations with gravity decades ago [46]. These configurations are adequate for a relativistic extension of the BEC model. From the action in Eq. (3), dimensionless time-independent Einstein and scalar wave equations appear as in Ref. 47: the (tt) and the (rr) components of Einstein equation are

$$\frac{A'}{A^2 x} + \frac{1}{x^2} \left[1 - \frac{1}{A} \right] - \left[\frac{\Omega^2}{B} + 1 \right] \sigma^2 - \frac{\Lambda}{2} \sigma^4 - \frac{\sigma'^2}{A} = 0 \quad (6)$$

and

$$\frac{B'}{ABx} - \frac{1}{x^2} \left[1 - \frac{1}{A} \right] - \left[\frac{\Omega^2}{B} - 1 \right] \sigma^2 + \frac{\Lambda}{2} \sigma^4 - \frac{\sigma'^2}{A} = 0, \quad (7)$$

respectively, while the equation of motion for the field is

$$\sigma'' + \left[\frac{2}{x} + \frac{B'}{2B} - \frac{A'}{2A} \right] \sigma' + A \left[\left(\frac{\Omega^2}{B} - 1 \right) \sigma - \Lambda \sigma^3 \right] = 0, \quad (8)$$

where $x = mr$, $\Omega = \frac{\omega}{m}$, $A \equiv [1 - 2\frac{M(x)}{x}]^{-1}$, $\phi(x, t) = (4\pi G)^{-\frac{1}{2}} \sigma(r) e^{-i\omega t}$, and $\Lambda = \frac{\lambda m_p^2}{4\pi m^2}$. In Refs. 8 and 9 the Newtonian v_{rot} was used while the relativistic rotation velocity includes the contribution from the pressure [32]:

$$v_{rot} = \sqrt{\frac{x B'(x)}{2B(x)}}. \quad (9)$$

Figure 1 shows rotation velocity curves for the case with $\Lambda = 300$. The parameters are $B(0) = 0.780$ and $\sigma(0) = 0.01$. Only the DM contribution is considered here. Including the visible matter changes the slope of the curves and explains well the variety of observed galaxy rotation curves [48]. Note that the density $\sim \sigma^2$ shows the core shape distribution.

For DM particles to be in the BEC state, they have to be gauge singlets, and the BEC phase transition must happen at the early universe. Regarding the origin of the DM scalar particles, it is interesting [20] that the observed rotation velocity, $v_{rot} \sim 10^{-3}$ (in units of c), corresponds to the central field strength of about GUT or inflation scale $\sigma(0) \sim v_{rot}^2 M_P$. This field value may be related to the BEC phase transition [49]. Figure 1 shows σ and rotation velocity curve of an 8 node solution ($n = 9$). The zero node solutions seem to be adequate for dwarf galaxies while higher-node solutions are required for larger galaxies.

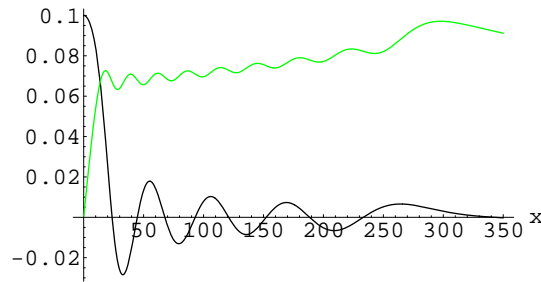


FIG. 1: Rotation velocity (green) and $10 \times \sigma$ as a function of position x for an 8 node solution. The parameters are $\Lambda = 300$, $\Omega = 0.9$, $B(0) = 0.780$ and $\sigma(0) = 0.01$.

For free-field case ($\Lambda = 0$) [50], it was found that there is a maximum mass $M_{max} = 0.636 \frac{M_P^2}{m}$ for the zero-node solution. For the case $\Lambda \neq 0$, a new scale appears because of the repulsive force preventing the halo from gravitational collapse. In this case, the typical length scale is $R \sim \Lambda^{\frac{1}{2}}/m$, thus, the typical mass scale is $\frac{R}{G} \sim \Lambda^{\frac{1}{2}} M_P^2/m$. A numerical study [47] shows $M_{max} = 0.236 \Lambda^{\frac{1}{2}} \frac{M_P^2}{m}$ for zero-node solutions. Note that $\Lambda = \lambda M_P^2/4\pi m^2$ is very large even for very small λ due to the smallness of m relative to M_P ; hence, the self-interaction effect is non-negligible. The Newtonian limits of the last equations for $\Lambda = 0$ lead to the non-linear Schrödinger equation for Sin's model [51].

A remaining concern about the BEC/SFDM model may be the stability of the excited states needed to explain flat rotation curves for large galaxies. It was shown [8, 9] that the DM halo is stable enough against gravitational radiation [52], and the evaporation and collapse procedure [8]. However, there are arguments [53] that the excited states are not stable against gravitational cooling [54]. This difficulty may be alleviated by considering the rotation of the halos. Finite angular momentum may prevent the collapse of the excited states, as is usual in an astronomical object. According to the bottom-up structure formation scenario of the Λ CDM model, large galaxies were made by the merging of smaller galaxies. Thus, it is plausible that the halos of the large galaxies got angular momentum during the merging process and have vortices within them [55].

Another merit of the BEC/SFDM model is that collisions of halos [56] can be described as BS collisions [57], which could open an intriguing new field of galaxy dynamics and evolution study. To do this, we need the Newtonian approximation of the Einstein equation and the scalar field equation

$$\begin{cases} \nabla^2 V = \sigma^2 + \frac{\Lambda \sigma^4}{4} \\ \nabla^2 \sigma = 2V\sigma \end{cases} \quad (10)$$

where V is the Newtonian gravity potential.

The BEC/SFDM model also provides us a fascinating possibility of simulating evolution of DM objects by using the results obtained from BEC experiments in a laboratory. It will also be interesting to study the cosmological effects of more general scalar fields [58].

In conclusion, since the BEC/SFDM model has passed test by test and seems to overcome the difficulties of heavy CDM particles, the future of this model seems to be very bright. Especially, if no WIMP is found at LHC or in other ground-based experiments soon, this model could be an interesting alternative.

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